**Assignment #3 – Solutions**

**(4.12,4.13,4.27,4.37,4.38,4.39)**

**4.12**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| ***t*** | **Day** | **Actual  Demand** | **Forecast  Demand** |  |
| 1 | Monday | 88 | 88 |  |
| 2 | Tuesday | 72 | 88 |  |
| 3 | Wednesday | 68 | 84 |  |
| 4 | Thursday | 48 | 80 |  |
| 5 | Friday |  | 72 | ← Answer |

*Ft* = *Ft–*1 + α(*At–*1 – *Ft–*1)

Let α = .25. Let Monday forecast demand = 88

*F*2 *=* 88 + .25(88 – 88) = 88 + 0 = 88

*F*3 *=* 88 + .25(72 – 88) = 88 – 4 = 84

*F*4 *=* 84 + .25(68 – 84) = 84 – 4 = 80

*F*5 *=* 80 + .25(48 – 80) = 80 – 8 = 72

**4.13** (a) Exponential smoothing, α = 0.6:

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | Exponential | Absolute |
| Year | Demand | Smoothing α = 0.6 | Deviation |
| 1 | 45 | 41 | 4.0 |
| 2 | 50 | 41.0 + 0.6(45–41) = 43.4 | 6.6 |
| 3 | 52 | 43.4 + 0.6(50–43.4) = 47.4 | 4.6 |
| 4 | 56 | 47.4 + 0.6(52–47.4) = 50.2 | 5.8 |
| 5 | 58 | 50.2 + 0.6(56–50.2) = 53.7 | 4.3 |
| 6 | ? | 53.7 + 0.6(58–53.7) = 56.3 |  |

Σ = 25.3

MAD = 5.06

Exponential smoothing, α = 0.9:

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | Exponential | Absolute |
| Year | Demand | Smoothing α = 0.9 | Deviation |
| 1 | 45 | 41 | 4.0 |
| 2 | 50 | 41.0 + 0.9(45–41) = 44.6 | 5.4 |
| 3 | 52 | 44.6 + 0.9(50–44.6 ) = 49.5 | 2.5 |
| 4 | 56 | 49.5 + 0.9(52–49.5) = 51.8 | 4.2 |
| 5 | 58 | 51.8 + 0.9(56–51.8) = 55.6 | 2.4 |
| 6 | ? | 55.6 + 0.9(58–55.6) = 57.8 |  |

Σ = 18.5

MAD = 3.7

(b) 3-year moving average:

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | Three-Year | Absolute |
| Year | Demand | Moving Average | Deviation |
| 1 | 45 |  |  |
| 2 | 50 |  |  |
| 3 | 52 |  |  |
| 4 | 56 | (45 + 50 + 52)/3 = 49 | 7 |
| 5 | 58 | (50 + 52 + 56)/3 = 52.7 | 5.3 |
| 6 | ? | (52 + 56 + 58)/3 = 55.3 |  |

Σ = 12.3

MAD = 6.2

c) Trend projection:

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  | Absolute |
| Year | Demand | Trend Projection | Deviation |
| 1 | 45 | 42.6 + 3.2 × 1 = 45.8 | 0.8 |
| 2 | 50 | 42.6 + 3.2 × 2 = 49.0 | 1.0 |
| 3 | 52 | 42.6 + 3.2 × 3 = 52.2 | 0.2 |
| 4 | 56 | 42.6 + 3.2 × 4 = 55.4 | 0.6 |
| 5 | 58 | 42.6 + 3.2 × 5 = 58.6 | 0.6 |
| 6 | ? | 42.6 + 3.2 × 6 = 61.8 |  |

Σ = 3.2

MAD = 0.64



|  |  |  |  |
| --- | --- | --- | --- |
| *X* | *Y* | *XY* | *X*2 |
| 1 | 45 | 45 | 1 |
| 2 | 50 | 100 | 4 |
| 3 | 52 | 156 | 9 |
| 4 | 56 | 224 | 16 |
| 5 | 58 | 290 | 25 |

Then: Σ*X =* 15, Σ*Y =* 261, Σ*XY =* 815, Σ*X*2 = 55, = 3, = 52.2 Therefore:



(d)  Comparing the results of the forecasting methodologies for parts (a), (b), and (c).

|  |  |
| --- | --- |
| Forecast Methodology | MAD |
| Exponential smoothing, α = 0.6 | 5.06 |
| Exponential smoothing, α = 0.9 | 3.7 |
| 3-year moving average | 6.2 |
| Trend projection | 0.64 |

Based on a mean absolute deviation criterion, the trend projection is to be preferred over the exponential smoothing with α = 0.6, exponential smoothing with α = 0.9, or the 3-year moving average forecast methodologies.

**4.27**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Winter | Spring | Summer | Fall |
| 2004 | 1,400 | 1,500 | 1,000 | 600 |
| 2005 | 1,200 | 1,400 | 2,100 | 750 |
| 2006 | 1,000 | 1,600 | 2,000 | 650 |
| 2007 | 900 | 1,500 | 1,900 | 500 |
|  | 4,500 | 6,000 | 7,000 | 2,500 |



**4.37** (a, b)

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Period | Demand | Forecast | Error | | Running sum | | |error| |
| 1 | 20 | 20 | 0.00 | | 0.00 | | 0.00 |
| 2 | 21 | 20 | 1.00 | | 1.00 | | 1.00 |
| 3 | 28 | 20.5 | 7.50 | | 8.50 | | 7.50 |
| 4 | 37 | 24.25 | 12.75 | | 21.25 | | 12.75 |
| 5 | 25 | 30.63 | –5.63 | | 15.63 | | 5.63 |
| 6 | 29 | 27.81 | 1.19 | | 16.82 | | 1.19 |
| 7 | 36 | 28.41 | 7.59 | | 24.41 | | 7.59 |
| 8 | 22 | 32.20 | –10.20 | | 14.21 | | 10.20 |
| 9 | 25 | 27.11 | –2.10 | | 12.10 | | 2.10 |
| 10 | 28 | 26.05 | 1.95 | | 14.05 | | 1.95 |
|  |  |  |  |  |  | MAD5.00 | |

RSFE = 14.05; MAD = 5 Tracking = 14.05/5 = 2.82

**4.38**(a) least squares equation: *Y* = –0.158 + 0.1308*X*

(b) *Y* = –0.158 + 0.1308(22) = 2.719 million

(c) coefficient of correlation = *r* = 0.966

coefficient of determination = *r*2 = 0.934

**4.39**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Year *X* | | Patients *Y* | | | *X*2 | | | *Y*2 | | | *XY* | | |
| 1 | | 36 | | | 1 | | | 1,296 | | | 36 | | |
| 2 | | 33 | | | 4 | | | 1,089 | | | 66 | | |
| 3 | | 40 | | | 9 | | | 1,600 | | | 120 | | |
| 4 | | 41 | | | 16 | | | 1,681 | | | 164 | | |
| 5 | | 40 | | | 25 | | | 1,600 | | | 200 | | |
| 6 | | 55 | | | 36 | | | 3,025 | | | 330 | | |
| 7 | | 60 | | | 49 | | | 3,600 | | | 420 | | |
| 8 | | 54 | | | 64 | | | 2,916 | | | 432 | | |
| 9 | | 58 | | | 81 | | | 3,364 | | | 522 | | |
| 10 | | 61 | | | 100 | | | 3,721 | | | 610 | | |
| 55 |  |  | 478 |  |  | 385 |  |  | 23,892 |  |  | 2,900 |  |

Given: *Y* = *a* + *bX* where:



and Σ*X* = 55, Σ*Y* = 478, Σ*XY* = 2900, Σ*X*2 = 385, Σ*Y*2 = 23892,  
 Then:



and *Y* = 29.76 + 3.28*X*. For:



Therefore:

Year 11 → 65.8 patients

Year 12 → 69.1 patients

The model “seems” to fit the data pretty well. One should, however, be more precise in judging the adequacy of the model.  
Two possible approaches are computation of (a) the correlation coefficient, or (b) the mean absolute deviation. The correlation coefficient:



The coefficient of determination of 0.853 is quite respectable—indicating our original judgment of a “good” fit was appropriate.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Year | Patients | Trend |  | Absolute |
| *X* | *Y* | Forecast | Deviation | Deviation |
| 1 | 36 | 29.8 + 3.28 ×  1 = 33.1 | 2.9 | 2.9 |
| 2 | 33 | 29.8 + 3.28 ×  2 = 36.3 | –3.3 | 3.3 |
| 3 | 40 | 29.8 + 3.28 ×  3 = 39.6 | 0.4 | 0.4 |
| 4 | 41 | 29.8 + 3.28 ×  4 = 42.9 | –1.9 | 1.9 |
| 5 | 40 | 29.8 + 3.28 ×  5 = 46.2 | –6.2 | 6.2 |
| 6 | 55 | 29.8 + 3.28 ×  6 = 49.4 | 5.6 | 5.6 |
| 7 | 60 | 29.8 + 3.28 ×  7 = 52.7 | 7.3 | 7.3 |
| 8 | 54 | 29.8 + 3.28 ×  8 = 56.1 | –2.1 | 2.1 |
| 9 | 58 | 29.8 + 3.28 ×  9 = 59.3 | –1.3 | 1.3 |
| 10 | 61 | 29.8 + 3.28 × 10 = 62.6 | –1.6 | 1.6 |
|  |  |  |  | Σ = 32.6 |
|  |  |  |  | MAD = 3.26 |

The MAD is 3.26—this is approximately 7% of the average number of patients and 10% of the minimum number of patients. We also see absolute deviations, for years 5, 6, and 7 in the range   
5.6–7.3. The comparison of the MAD with the average and minimum number of patients and the comparatively large deviations during the middle years indicate that the forecast model is not exceptionally accurate. It is more useful for predicting general trends than the actual number of patients to be seen in a specific year.